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ABSTRACT

Classical time-invariant lens aberrations, and methods for correcting them, are well known in the art. However, the design, analysis, and construction of optical components and systems for temporally modulated optical wavefronts–and in particular, wavefronts in optical time-of-flight or phase measurement instruments, such as laser trackers, heterodyne laser interferometers, and spherical retroreflectors, require additional considerations to correct for what will be called Optical Amplitude Modulation (OAM) aberrations. Ray tracing analysis is time-invariant and thus insensitive to temporal modulation of the rays. Secondary considerations must be given to the wavefront of the modulated envelope which is focused on a detector, i.e., while the rays converge to a focus, the phase of the modulated envelope will in general depend on the path of the rays. Elements from communications theory, including amplitude modulation (AM) and analysis in the Fourier transform frequency domain are unified with classical optics, where the optical wavelength of a laser is treated as a carrier signal and the AM produces two slightly offset sidebands. The sidebands produce the OAM aberration due to dispersion and different paths through the optical elements. Suggestions are made for methods for correcting OAM aberrations, such as lens designs that are achromatic at the two sidebands, the use of specific materials matched to the carrier wavelength, the use of corrector plates, and computer modeling tools. A review of relevant patent literature is included.

Keywords: optics design software, ray tracing, dispersion, aberration, optical amplitude modulation, electronic distance measurement, laser tracker, total station

1. INTRODUCTION

This paper is related to a family of US patents by the author (The Patent Family); *Methods and apparatus for optical amplitude modulated wavefront shaping*,[1] and *Methods for modeling amplitude modulated light through dispersive optical systems and electronic distance measurement instruments*.[2] The patents include a lengthy explanation of how dispersion affects the propagation of amplitude modulated light through an optical system, from first principles, i.e., using Fourier optics techniques[3–6] and communications theory.[7–9] This lengthy explanation will not be repeated in this paper.

Unfortunately, the contractor that typesets the patent publications for the US Patent and Trademark Office (USPTO), has problems properly typesetting more complex equations, and since the official documents are the original documents filed in the application–which are preserved and publicly available–it is USPTO policy not to correct publication errors that do not directly relate to the patent claims. The net result is that the derivations published in the patent publications are fraught with errors, and hard to read. However, the original specification, produced in LaTeX, is available on our website `www.parker-ip-ent.com` and in the USPTO Public Patent Application Information Retrieval (Public PAIR) system at `https://portal.uspto.gov/pair/PublicPair`.

Time-invariant aberrations due to dispersion are well known in the art.[10–17] The technical literature has a rich heritage of rules of thumb, first order approximations, technical nomenclature, and fabrication techniques. Optical systems can be designed based on published manufacturer's specifications.

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Most optical systems are time-invariant systems that produce an image on an integrating detector, such as film or a CCD sensor. When people talk about modulation in such systems, they are typically talking about spatial modulation, such as Ronchi patterns.

However, there are nonimaging optical systems that focus amplitude modulated light onto analog detectors that receive the modulated light and recover the modulation signal–much like AM radio uses a higher frequency carrier wave to transmit audio information. Systems that use light as a carrier, and temporally modulate the amplitude, power, phase, polarization, frequency, etc., modulated at radio frequency (RF), microwave (MW), or terahertz (THz) frequencies in the time domain, so as to produce optical sideband frequencies that propagate at different velocities in a dispersive medium, will be called Optical Amplitude Modulation (OAM) systems. Aberrations in OAM systems that produce errors in the recovered modulated signal will be called OAM aberrations.

With modern computers, optical design software, and measuring and testing instrumentation, it is now common to correct for higher order time-invariant aberrations and to optimize designs involving many degrees of freedom, including choices of materials, optical element geometry, and systems architectures to produce doublet, multielement, aspherical, and achromatic elements.

For example, there are a number of excellent computer modeling tools available for analysis and engineering of optical elements and systems, such as: Beam 4, available from Stellar Software; CODE-V, available from Synopsys; FRED, available from Photon Engineering; Optica 4, which runs under the Mathematica environment, available from Optica Software; ZEMAX, available from Zemax LLC; and others. There are a number of US Patents which disclose methods for modeling optical systems, some of which are cited in the bibliography.[18–36]

Some of these tools do use analysis of modulation in the spatial domain, such as the modulation transfer function (MTF), to model such things as the capability of an optical system to resolve line pairs. However, based on the product literature and patent disclosures, none of these tools are designed to work with RF, or higher frequency, temporally modulated wavefronts, in a dispersive medium, i.e., they are designed for time-invariant applications.

Optical test methods are well known in the art for imaging optical components. For example, *Optical Shop Testing*, Daniel Malacara[13] is a comprehensive reference on the subject, including such topics as interferometry, Ronchi patterns, Moiré, etc. Low frequency phase modulation, and discrete steps in phase, are used in interferometry and Moiré, however these techniques are quasi-time-invariant and do not fit the criteria for OAM systems.

Other than suggestions made in The Patent Family,[1,2] there are no known standard testing methods for optical components as to OAM. Moreover, there are no known standard commercial catalog terms, nomenclature, or specifications to even describe OAM parameters of passive optical components, i.e., there isn't even a name for a lens designed for reduced OAM aberration.

One possible reason for this omission from the literature is that OAM systems are somewhat esoteric and heretofore relegated primarily to time-of-flight or phase measurement instrumentation, commonly called electronic distance measurement or electromagnetic distance measurement (EDM) instruments.[37–43] Another reason is that electro-optical system designs naturally break between the classical optics, and electronics systems disciplines; i.e., the optical engineer takes the optical design to the detector, and the electronics engineer picks it up from the detector. Subtle effects in the optics due to the modulation, such as dispersion, can easily be missed.

In the case of high precision distance measurements, such as with laser interferometers and laser trackers, there are no other instruments to compare the results against, so the errors can go totally undetected. Even when such subtle errors are observed, it can be easy to dismiss them as random experimental errors–much like the radio astronomers that saw, but dismissed, the errors due to the cosmic microwave background, until Penzias and Wilson pursued an explanation of the source.

2. DISPERSION

Most optics books, and even introductory level physics books covering sound and quantum mechanics, give a brief explanation of the superposition, or interference, of waves. The underlying information in a superposition of waves is commonly described as the beat note, envelope, packet, or group. For example, chapter 12 of Jenkins and White[11] gives an example of the superposition of waves of slightly different frequencies and derives the group velocity of a wave packet. Equation 12p of Jenkins and White derives the group velocity u as

$$u = v - \lambda \frac{dv}{d\lambda} \tag{1}$$

where v is the wave velocity and λ is the wavelength, in the medium, i.e., they change at an air/glass interface.

Rüeger[38] rewrites equation 1 in terms of the group refractive index n_g, the phase refractive index n, and wavelength λ, in his equation 5.11

$$n_g = n - \frac{dn}{d\lambda}\lambda \tag{2}$$

where

$$n = \frac{c}{v} \tag{3}$$

$$n_g = \frac{c}{u} \tag{4}$$

and c is the speed of light. He further defines group refractivity N_g in equation 6.13, for convenience, as

$$Ng = (n_g - 1)10^6. \tag{5}$$

He shows example calculations of n_g for air at various wavelengths, temperatures, water vapor pressures, and atmospheric pressures, as well as table 10.2 for BK7 glass at various wavelengths. Additional details are included in his Appendix A. Example calculations, including a Mathematica program to calculate n_g, can also be found in Green Bank Telescope Memo 230.[44]

While equations 1 and 2 describe how the group velocity u and group refractive index n_g are calculated, it doesn't give one an intuitive understanding of the physics involved. Some examples will help in understanding the physics, thus developing an intuitive understanding.

For most glass optical materials, the refractive index decreases for longer wavelengths, i.e.,

$$\frac{dn}{d\lambda} < 0 \tag{6}$$

$$n_g > n. \tag{7}$$

As shown in **FIG. 1**, a 10 Hz carrier (not shown) modulated by a 0.5 Hz modulating signal, generates an upper sideband at 10.5 Hz, and a lower sideband at 9.5 Hz. The power of the beat note is produced by the square of the sum of the upper and lower sidebands. As shown in figure 12I of Jenkins and White,[11] the wavelength Λ of the beat note is

$$\Lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \tag{8}$$

where λ_1 is the wavelength of the lower sideband and λ_2 is the wavelength of the upper sideband, in the medium, i.e., the wavelengths are different in glass and air. Note that the frequency of the power is the second harmonic of the 0.5 Hz modulating frequency, i.e., 1 Hz.

In **FIG. 1**, the phase angle between the lower and upper sidebands, at time = 0, is 0. **FIG. 2** shows the same functions, except the upper sideband is delayed in phase, at time = 0, by π.

Notice that the relative phase shift π between the lower and upper sidebands produces the same phase shift π in the beat note. This is a fundamental principle that warrants further illustration.

Consider a gedanken, or thought, experiment. In a first case, imagine a ray of light comprising the upper sideband at frequency $f + \Delta f$ and lower sideband at frequency $f - \Delta f$ is passing through a vacuum with no dispersion, i.e., $n = n_g = 1.000\,000$ and the lower sideband travels at exactly the same speed v_l as the upper sideband v_u, $v_l = v_u = c$. Since there is no dispersion, the group velocity u is also c. Starting at time = 0, a detector measures the beat note, as shown in **FIG. 1**.

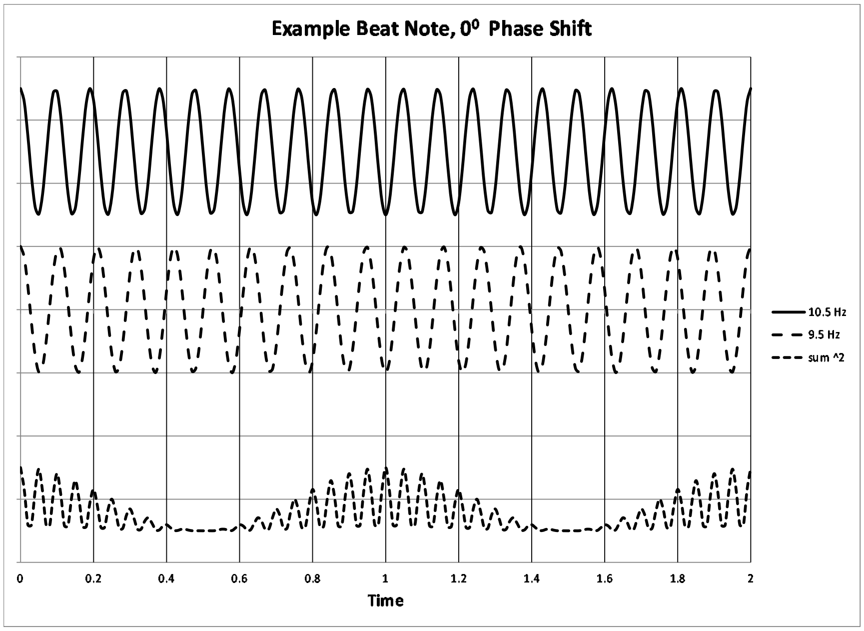

Figure 1. Beat note produced by two sidebands.

In a second case, imagine the same conditions as the first case, except that somehow the upper sideband is shifted by π. The detector would measure the beat note also shifted by π, as shown in **FIG. 2**. By shifting the relative phase between the upper sideband and the lower sideband by a half wavelength, which only requires delaying the upper sideband by a fraction of a micron, the beat note–which has a much longer wavelength as shown in equation 8–is also apparently delayed by a half wavelength.

In an EDM instrument, the group wavelength is typically of the order of 100 mm. For an accuracy of the order of 1 μm, or 10 parts per million of a wavelength, the corresponding differential phase shift would be $2\pi \times 10^{-5} = 6.3 \times 10^{-5}$ radians. This is why an infinitesimal phase shift, introduced by dispersion, is so important for EDM applications.

In a third case, imagine a 50 mm thick glass window is inserted in the path of the ray. Assume the glass has no dispersion and at the upper sideband $n_u = 1.500\,000$ and at the lower sideband $n_l = 1.500\,000$. Now, one has a more interesting situation to analyze, due to two discontinuities at the entrance and exit faces of the window.

At the entrance face, the refractive index changes from $n = 1.000\,000$ to $n_u = n_l = 1.500\,000$. The frequencies of the upper sideband and lower sideband are invariant, but the wavelengths of the upper sideband and the lower sideband change while propagating through the glass.

$$\lambda_{upper} = \frac{c}{n_u(f + \Delta f)} \quad (9)$$

$$\lambda_{lower} = \frac{c}{n_l(f - \Delta f)}. \quad (10)$$

Since the refractive indexes are the same, the wavelengths change by the same scale factor. At the exit face, where $n = 1.00\,000$ again, the relative phases between the upper and lower sidebands remain the same, i.e., they are the same as if the sidebands traveled through $1.500\,000 \times 50$ mm, or an extra 25 mm, of vacuum. The

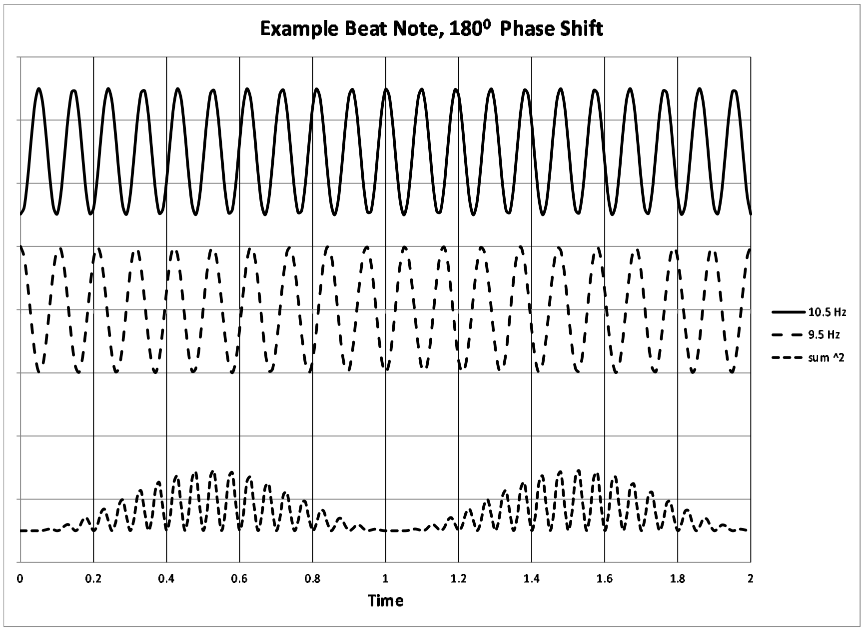

Figure 2. Beat note produced by two sidebands, shifted by 180°.

detector would measure a first phase shift in the beat note due to the delay introduced by the slower propagation speed through the glass.

In a fourth case, imagine the 50 mm glass window is replaced by another 50 mm glass window with dispersion and at the upper sideband $n_u = 1.500\,010$ and at the lower sideband $n_l = 1.500\,000$. Unlike the third case, now the wavelengths do not change by a simple scale factor. The wavelength of the upper sideband is actually stretched slightly longer than in the third case. At the exit face, compared to the third case, the upper sideband is slightly retarded with respect to the lower sideband. This slight extra phase shift produces the same slight extra phase shift in the group. The detector would measure a second phase shift, which is significantly different from the first phase shift–due to the dispersion.

In a fifth case, imagine the 50 mm thick dispersive glass window is replaced by a 40 mm thick window of the same dispersive material. The detector would measure a third phase shift which is significantly different from the second phase shift–due to the difference in thickness.

3. WAVEFRONT OF A MODULATED SIGNAL THROUGH OPTICAL ELEMENTS

Turning now to some examples, it will be shown how inhomogeneities in the OAM wavefront of the beat note can produce errors in EDM instruments. One may argue that since the optical elements in a system remain constant, the integrated sum of the phase shifts due to dispersion are a fixed constant and can be ignored. However, as will be shown, the fallacy of that argument is that the beam profile is subject to different conditions which makes the integrated sum of the phase shifts due to dispersion vary.

3.1 Lens

Consider an ideal converging lens **51**, as shown in **FIG. 3**, designed to focus an object at infinity to a plane at a distance equal to the focal length f **52**, i.e., designed for infinite conjugate ratio. While the example is for

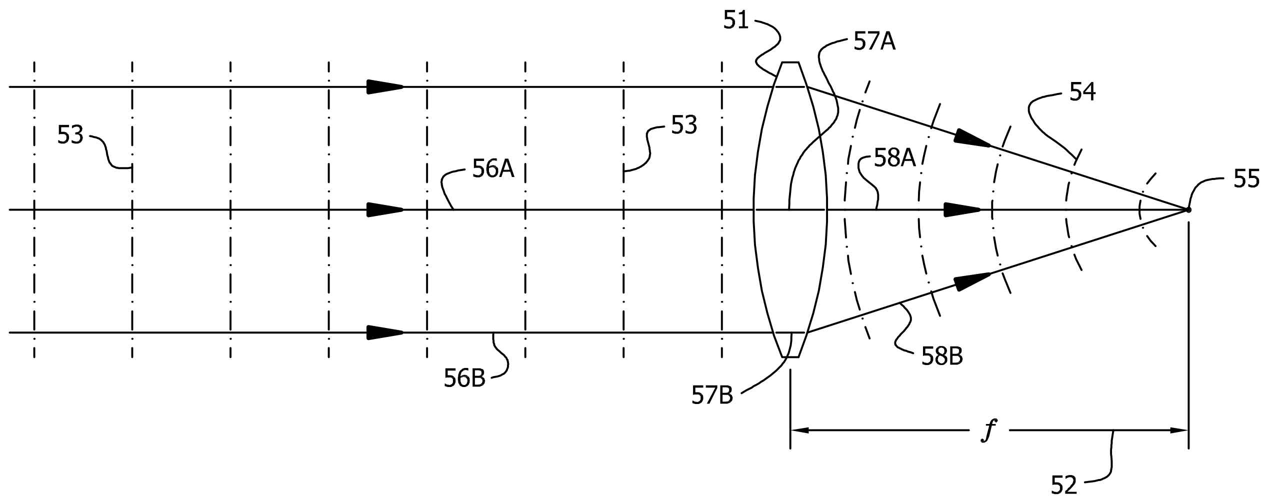

Figure 3. Plane wave and converging lens.

infinite conjugate ratio, the same principles apply to other conjugate ratios, or adjustable conjugate ratios–such as zoom lenses.

A monochromatic coherent light source at infinity produces a plane wave **53** at the lens **51**, and the lens **51** converts the plane wave **53** to spherical waves **54** which focuses the plane wave **53** to a point **55** at the focal length **52**. Lenses are routinely designed, built, and tested that perform this function with wavefronts flat to an accuracy of a fraction of the design wavelength λ.

In a simple explanation, the transformation of a lens can be explained by Fermat's principle which, in this example, requires the transit time to be equal for all rays from the source, through the lens, to the focal point. In the ideal case, all of the rays from a coherent source would converge at the focal point in phase. From Fermat's principle, a ray passing through the optical axis **56A, 57A, 58A** must be delayed through the lens **51** longer than a ray passing through the edge **56B, 57B, 58B** of the lens **51**, since the ray passing through the edge **56B, 57B, 58B** of the lens **51** must travel farther to the focal point **55**. Thus the lens, which has a higher refractive index for glass $n_{(glass)}$ than the refractive index of air $n_{(air)}$, is thicker in the center and thinner at the edges.

The time τ for a ray passing through the center of the lens, through distances $\overline{56A}$, $\overline{57A}$, and $\overline{58A}$ is the same as for a ray passing through the edge of the lens, through distances $\overline{56B}$, $\overline{57B}$, and $\overline{58B}$. This can be written as

$$\tau = 1/c\{n_{(air)}(\overline{56A}+\overline{58A})+n_{(glass)}\overline{57A}\} \quad (11)$$

$$= 1/c\{n_{(air)}(\overline{56B}+\overline{58B})+n_{(glass)}\overline{57B}\} \quad (12)$$

or simplified to

$$n_{(air)}(\overline{56A}+\overline{58A})+n_{(glass)}\overline{57A}=n_{(air)}(\overline{56B}+\overline{58B})+n_{(glass)}\overline{57B}. \quad (13)$$

However, in general, this is not the case for the modulated wavefront that propagates at the group velocities for the group refractive indexes for air $n_{g(air)}$ and glass $n_{g(glass)}$. In other words,

$$n_{g(air)}(\overline{56A}+\overline{58A})+n_{g(glass)}\overline{57A}\neq n_{g(air)}(\overline{56B}+\overline{58B})+n_{g(glass)}\overline{57B}. \quad (14)$$

The net result is that the phase of the modulation group of a ray passing through the center of the lens will be delayed reaching the focal point **55** more than the phase of the modulation group of a ray passing through the edge of the lens.

This is analogous to fading in radio signals due to multipath induced fading–which not only changes the amplitude of the signal, but also the phase. For EDM instruments, this is an error in the measured distance.

This has many ramifications. For example, if the beam is stopped down and outer rays are blocked, the measured distance could be too short. The measured distance could depend on the aperture of the retroreflector. Contamination, such as dust or fingerprints, on an optical element may produce scatter, thus reducing power for a portion of the beam.

3.2 Spherical Retroreflector

Some instruments use spherical, also called cat's eye, retroreflectors.[45–58] Some designs use specialty glass that has a refractive index of 2.0, formed in a single sphere. Other designs use concentric spherical shells of different glass materials. Another design uses two hemispheres of the same material, but different radiuses, glued together. In all cases, rays passing through the center of the retroreflector and parallel rays passing through the retroreflector offset from the axis are retroreflected.

The designs are illustrated with ray tracings, but none of the designs address the phase of the modulated signal being retroreflected. Yet, like the lens example above, the rays pass through different thicknesses of glass, depending on the offset from the optical axis. For EDM instruments, this is an error in the measured distance.

3.3 Flat Plate in Converging Beam

Similar problems are exhibited when a flat plate, or beam splitter, is inserted in a converging beam, i.e., the beam that passes through at an angle passes through more glass than the beam normal to the plate.

4. INSTRUMENTS SUBJECT TO OAM ABERRATIONS

In general, any instrument that uses two frequencies is subject to OAM aberrations. EDM instruments typically generate upper and lower sidebands by modulating a carrier;[59–74] or by generating two frequencies by Zeeman splitting or locking two lasers having different frequencies.[75–87]

Radio astronomy uses interferometry between arrays of radio telescopes to produce interference fringes of astronomical sources.[43] Pulsar sources produce amplitude modulated signals due to rapidly rotating sources. While radio telescopes use reflective optics, which are not subject to dispersion, for main reflectors, some receivers do use microwave lenses to correct for aberrations of the main reflector. The microwave lenses are subject to OAM aberrations, and should be modeled.

In particular, correction of OAM aberrations would be useful for the highest accuracy EDM instruments, such as laser interferometers and laser trackers. Laser trackers are manufactured by API Automated Precision, FARO Technologies, Leica Geosystems, and Nikon Metrology. Reviews of laser tracker technology and applications can be found in Muralikrishnan et al.,[88] and Parker.[89,90]

5. METHODS FOR CORRECTING OAM ABERRATIONS

As suggested in The Patent Family,[1,2] there are techniques that are used to correct phase wavefronts that could possibly be used to correct OAM aberrations.[91–93] A simple way to avoid dispersion problems is to use reflective optics for precision instruments. This may not always be practical, but should at least be considered.

Some optical materials have resonances which produce local maxima and minima in the refractive index.[94] If an optical material can be made, or found, with local maxima or minima in the refractive index matched to the carrier wavelength, there would be no dispersion for the sidebands. In principle, lenses could be designed for specific laser wavelengths that are OAM aberration free.

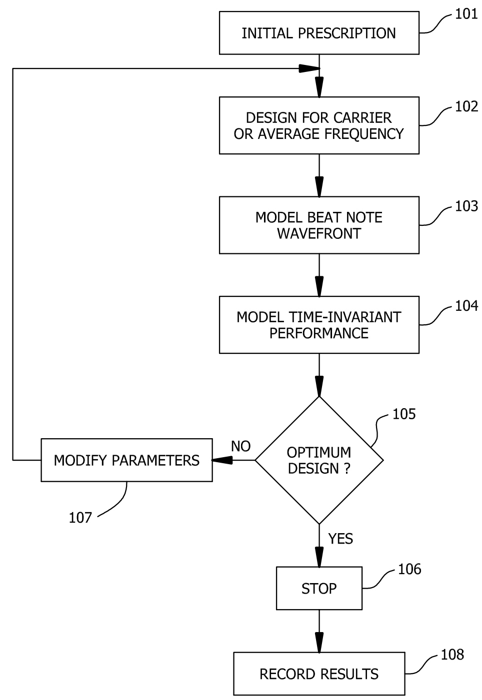

Figure 4. Flow chart to optimize conventional carrier frequency wavefront and new feature to also optimize beat note wavefront.

5.1 Optical Design Software

The first thing that is needed is optical design software that properly models OAM aberrations and wavefront distortions of the modulation group.

Smith[14] describes the synthesis of optical system design in Chapter 16. He describes one process as an optimization process where the change in aberration ΔA_n is described in his Eq. 16.15

$$\Delta A_n = \sum_{i=1}^{i=k} \left(\frac{\delta A_n}{\delta C} \right)_i \Delta C_i \tag{15}$$

where ΔC_i is the change in parameter C_i required to produce a change and $(\delta A_n/\delta C)_i$ is the partial of the aberration A_n with respect to C_i.

In section 16.8, Smith describes the process.

> The computer is presented with an initial prescription and a set of desired values for a limited set of aberrations. The program then computes the partial differentials of the aberrations with respect to each parameter (curvature, spacing, etc.) which is to be adjusted, and establishes a set of simultaneous equations (Eq. 16.15), which it then solves for the necessary changes in the parameters. Since this linear solution is an approximate one, the computer then applies these changes to the prescription (assuming that the solution is an improvement) and continues to repeat the process until the aberrations are at the desired values. When there are more variable parameters than system characteristics to be controlled, there is no unique solution to the simultaneous equations; in this case, the computer will add another requirement, namely that the sum of the squares of the (suitably weighted) parameters changes be a minimum.

Presumably, something along these lines is how conventional optical design software works. However, there are no known products that include corrections for OAM aberrations. They would need to be modified slightly to include a ΔA_n term for OAM aberration. One possible method is shown in **FIG. 4**. The initial prescription **101** for the optical system would need to be defined. This would include such things as focal length, beam size, aperture size, wavelength, etc. The optical system could be designed using conventional design techniques for the carrier frequency, or average frequency of a band **102**. The conventional design could then be enhanced by including modeling of the beat note **103**. The design could also be modeled for the conventional time-invariant parameters **104**. The combination of models **103, 104** could then be evaluated for optimization **105**. If the design meets the criteria for time-invariant **104** and beat note **103** constraints, the process is complete **106**, If not, parameters are modified **107**, and the process is repeated until the process is complete **106** and the results are recorded **108**.

For example; for an EDM system, in addition to the time-invariant requirement for focusing the power to a point on the detector, the software could also add a term for a requirement such as

$$\Phi = \frac{\int_A \rho\phi \, da}{\int_A \rho \, da} \tag{16}$$

where Φ is the integrated phase of the signal at the detector weighted by the power; ρ is the optical power density, and ϕ is the phase of the beat note wavefront over area da, and A is the area of the aperture. Other requirements may be made for various design constraints. The ideal condition would be for ϕ to be invariant over any area da of the beam, i.e., for ϕ to be flat over the entire beam.

A possible design criterian could include the condition that

$$d\Phi/da < \epsilon \tag{17}$$

where ϵ is an acceptable OAM aberration. Another possible design criterian would be to assume a beam profile for ρ, such as a Gaussian with a specified full width at half maximum, or a flat profile. The standard deviation σ of ϕ for uniformally sampled regions over the area A of the lens, for selected carrier frequencies and modulation frequencies, would be a number that could be specified for a standard catalog item lens that would be helpful to a designer. For example, knowing the σ for common laser carrier frequencies and various modulation frequencies for a stock lens would be very useful in the design of an optical system.

5.2 Design Data

Of course for the design software to be most useful, optical element manufacturers need to provide the relevant specifications to the design software.

6. CONCLUSIONS

It has been shown that there is an opportunity to improve optical systems that employ amplitude modulation of light, or mixing of closely matched sidebands, by properly analyzing heretofore overlooked OAM aberrations.

In order to enable such an analysis there is a need for optical design software that incorporates analysis of the modulated wavefront. Since there are no known commercial design software packages that include these capabilities, this presents an opportunity for enhancements to existing software packages and opportunities for new companies to enter the market with a unique capability.

The subject of OAM aberration and optical system presents a new field of optics research and development, and intellectual property, opportunities for the academic community, software designers, optical systems designers, manufacturing companies, and instrument companies.

Parker Intellectual Property Enterprises, LLC would like to work with those interested in the subject through the sale, or licensing, of The Patent Family,[1,2] or assisting others in filing patents on new inventions.

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